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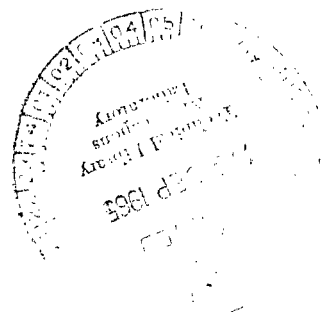


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ACCELERATION OF A CONDUCTING FLUID BY A TRAVELING MAGNETIC FIELD

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Cleveland, Ohio*





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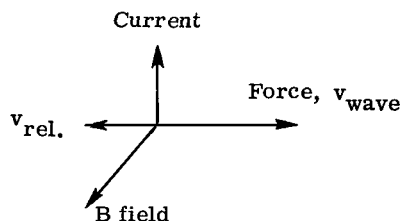
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SUMMARY

An analysis is undertaken for the traveling-wave accelerator with the propellant having scalar conductivity. Two configurations of magnetic field are studied. The first employs a field perpendicular to the line of motion and varying sinusoidally. When the conductivity is assumed proportional to the density, the analysis shows that the propellant is ejected in dense pulses, there is an upper limit on efficiency of about one-half, and the acceleration to wave velocity can be performed in one magnetic field wavelength if a defined interaction parameter has a value of order one. Another accelerator using a magnetic wave such as might be produced by coils surrounding the channel has a somewhat lower efficiency, which is due mainly to the loss of propellant to (or through) the boundaries.

INTRODUCTION

The desire for high specific impulses for the propulsion of interplanetary vehicles has led to research on several types of electromagnetic rockets. One of these is the traveling-magnetic-wave accelerator, which uses the principle of the induction motor. In this device a moving magnetic field passing through a conducting gas tends to drag the gas along with it. The force exerted on the gas depends on the velocity of the gas relative to the magnetic field (see sketch). Only when the gas has reached wave velocity will the acceleration process stop; thus, the maximum impulse that can be imparted to the gas is determined by the wave speed.



Several types of analysis have been made of devices of this kind. The studies of Klein and Brueckner (ref. 1) and Matthews (refs. 2 and 3) consider the coils that establish the magnetic field to be pulsed very rapidly in succession so that the propellant is accelerated by being

pinched by each coil in turn. Treatments by Meyer (ref. 4) and Covert and Haldeman (ref. 5) regard the device as a pump that produces a steady flow against a pressure gradient. Fabri and Moulin (ref. 6) and Bernstein, et al. (ref. 7) have linearized the equations of motion for small differences between wave and fluid speeds.

This report attempts to preserve the nonlinear character of the equations of motion during an acceleration process that takes the fluid from nearly zero velocity to nearly wave velocity. Several simplifying assumptions about the properties of the fluid are made to make the analysis tractable. The analysis is applied first to an accelerator with the magnetic field normal to the axis of motion and operating at a high enough density that the conductivity is a scalar; these are conditions characteristic of the experimental accelerator described by Heflinger and Schaffer (ref. 8). Secondly, there is a discussion of the accelerator with a magnetic field configuration similar to that of the device reported by Jones and Palmer (ref. 9), in which the magnetic wave has both longitudinal and transverse components.

SYMBOLS

A	magnetic vector potential, Wb/m
a	initial coordinate of fluid particle, m
B	magnetic field, Wb/sq m
B_s	steady uniform magnetic field, Wb/sq m
C	cross section of accelerator, sq m
c	speed of light, m/sec
d	accelerator width, m
F	instantaneous force on propellant, N
J	conduction current, A
j	current density, A/sq m
k	wave number, 1/m
p	pressure, N/sq m
R	gas constant, J/(kg)($^{\circ}$ K)
r	velocity ratio, v_o/v_w
T	temperature, $^{\circ}$ K
T_a	average temperature, $^{\circ}$ K

t	time, sec
t_p	time at which a density peak occurs, sec
u	dummy variable
v	velocity of fluid particle, m/sec
v_w	velocity of magnetic wave, m/sec
X	length of accelerator, m
x	longitudinal coordinate, m
y	transverse coordinate, m
z	dummy variable
\hat{z}	unit vector in z-direction
α	dimensionless initial coordinate, ka
β	magnetic field ratio, B_s/B_o
ϵ	perturbation parameter, RT_a/v_w^2
ζ	dimensionless transverse coordinate, ky
η	efficiency, eq. (38)
Θ	thrust, N
κ	interaction parameter, $\sigma_o B_o^2/\rho_o \omega$
λ	wavelength of magnetic wave, m
μ	permeability, H/m
ξ	dimensionless longitudinal coordinate, kx
ρ	density, kg/cu m
σ	conductivity, mho/m
τ	dimensionless time, ωt
φ	phase of magnetic wave, $kx - \omega t$
ω	angular frequency of magnetic wave, rad/sec

Subscripts:

o	initial value
w	wave
x, y	vector components

τ, α partial differentiation with respect to τ, α

Superscripts:

0, 1 zero and first order in perturbation process

average

CONFIGURATION

As a simple, idealized model of the traveling-wave accelerator, consider a two-dimensional channel of infinite extent in the x-direction (fig. 1). An alternating wave of magnetic induction, which is assumed to be uniform across the channel, moves from left to right with speed v_w . Fluid enters the channel from the left at the density ρ_0 and speed v_0 . When the fluid reaches the origin the magnetic field begins to act upon it, either because it has been suddenly made electrically conducting or because it has been previously shielded from the field. During the acceleration of the fluid by the field, currents flow in a direction normal to the plane of the motion and the field. The direction of these currents reverses as the fluid moves through a region in which the field reverses direction. The problem of closing the current loops at the sides of the accelerator is ignored here as well as the problem of the flow at the exit. If the two-dimensional accelerator is regarded as the limiting case of an annular accelerator as the two radii increase without limit, then the currents form closed loops about the axis and there is no problem of current-loop closure at the sides.

MAGNETIC WAVE

The equation that governs the vector potential $\hat{z}A$ of a magnetic field B in the x, y-plane (fig. 1) is

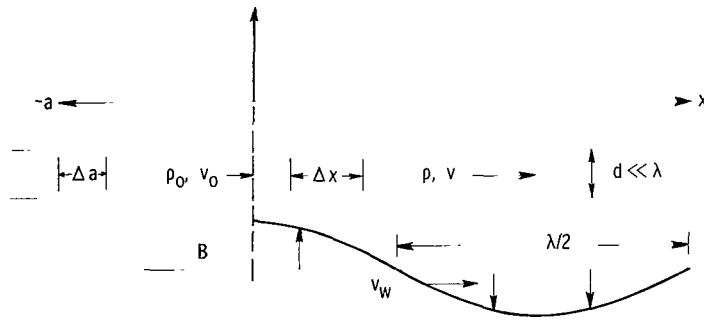


Figure 1. - Schematic of traveling-wave accelerator.

$$\nabla^2 A - \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} = -\mu J$$

For a quasi-static field of wave-like form, the differential operators occurring in the equation have magnitudes specified by

$$\nabla^2 \sim \lambda^{-2}$$

$$\frac{\partial^2}{\partial t^2} \sim \left(\frac{\omega}{2\pi} \right)^2 = \left(\frac{v_w}{\lambda} \right)^2$$

The quantities A and J have the magnitudes

$$A \sim B\lambda$$

$$J \sim \sigma v_w B$$

so that the three terms in the equation for A have the relative magnitudes 1 , $(v_w/c)^2$, and $\sigma\mu v_w\lambda$. Inasmuch as the desired magnitude of the wave velocity is $v_w \sim 10^5$ meters per second and the speed of light is $c = 3 \times 10^8$ meters per second, the second term has a relative magnitude 10^{-6} and can be safely ignored. The third term is a magnetic Reynolds number based on the wave speed and length. It is assumed that the analysis is restricted to small values of this number:

$$\sigma\mu v_w\lambda \ll 1 \tag{1}$$

It follows from these considerations that the vector potential is determined by

$$\nabla^2 A = 0$$

If A is required to have a wave-like character,

$$A = f(y)g(x - v_w t)$$

a solution that is symmetric about the x -axis is

$$A = \frac{B_0}{k} \cosh ky \cos(kx - \omega t) \quad (2)$$

For the case of a narrow channel with

$$\frac{d}{\lambda} \ll 1$$

it is nearly correct to write

$$A = B_0 k^{-1} \cos(kx - \omega t) \quad (3)$$

Equation (2) represents the kind of magnetic field first introduced for traveling-wave machines by R. X. Meyer (ref. 4).

LAGRANGIAN EQUATIONS OF MOTION

Rather than to view this problem as the study of a velocity field, it proves to be convenient to follow each fluid element separately - the so-called Lagrangian viewpoint. Although both approaches are fully described only by a set of nonlinear partial differential equations, with suitable approximations the Lagrangian equations of motion can be reduced to a set of ordinary differential equations. Averages over all the fluid elements then yield mean values for the accelerator thrust, efficiency, etc.

The Lagrangian equations of motion (Lamb, ref. 10) for a one-dimensional magneto-hydrodynamic flow are as follows:

Continuity:

$$\frac{\partial}{\partial t} \left(\rho \frac{\partial x}{\partial a} \right) = 0 \quad (4)$$

Momentum:

$$\rho \frac{\partial^2 x}{\partial t^2} + \frac{\partial p}{\partial x} = \sigma \left(v_w - \frac{\partial x}{\partial t} \right) B^2 \quad (5)$$

The continuity equation contains a derivative with respect to the distance a , which represents the original position of the fluid element, and, therefore, acts as a label to distinguish fluid elements throughout their motions. It is convenient to take $-\infty \leq a \leq 0$

so that the left semiaxis (fig. 1) may be regarded as occupied by a uniform fluid that at time $t = 0$ is moved at constant speed v_0 into the accelerating chamber $x \geq 0$. If C is the cross section of the accelerator, then the mass associated with fluid elements in the incremental distance Δa is $\rho_0 C \Delta a$. During the acceleration this same mass is represented by $\rho C \Delta x$. Thus, taking the limit as Δa becomes very small yields

$$\frac{\rho_0}{\rho} = \frac{\partial x}{\partial a} \quad (6)$$

which is a statement equivalent to equation (4) along with its initial condition.

The momentum equation (5) shows that the Lorentz force leads to both an acceleration and a compression of the fluid. The velocity $(v_w - \partial x / \partial t)$ is the relative velocity between the fluid and the wave. At the point where the fluid enters the accelerating chamber the relative velocity is greatest, and therefore the force on the fluid is maximum. Later, as the fluid velocity approaches wave velocity, the force diminishes and the fluid coasts along with the field.

NONDIMENSIONAL EQUATIONS OF MOTION

Let the dimensionless independent variables be defined as

$$\left. \begin{aligned} \frac{2\pi a}{\lambda} &\equiv ka = \alpha \\ \omega t &\equiv \tau \end{aligned} \right\} \quad (7)$$

and the dependent variable as

$$\xi(\alpha, \tau) \equiv kx(a, t) \quad (8)$$

The introduction of equations (3) and (6) to (8) into the momentum equation (5) yields

$$\frac{\partial^2 \xi}{\partial \tau^2} + \frac{p}{\rho v_w^2} \frac{\rho}{\rho_0} \frac{\partial}{\partial \alpha} \ln \frac{p}{p_0} = \frac{\sigma B_0^2}{\rho \omega} \left(1 - \frac{\partial \xi}{\partial \tau} \right) \sin^2 (\xi - \tau) \quad (9)$$

In a practical accelerator it is necessary that the terms representing the Lorentz force and the inertia force have the same order of magnitude. Thus, since the derivatives of ξ are of order unity, it is necessary that

$$\frac{\sigma B_o^2}{\rho \omega} \sim 1 \quad (10)$$

Combined with the provision of small magnetic Reynolds number (1), this condition requires

$$\frac{\rho v_w^2}{B_o^2/\mu} \ll 1 \quad (11)$$

Roughly,

$$\rho v_w \sim \rho_o v_o$$

so that equation (10) becomes

$$\frac{\rho_o v_o v_w}{B_o^2/\mu} \ll 1 \quad (12)$$

The condition (12) on the parameters of the accelerator requires that the flux of momentum in the fluid be very small compared to the magnetic pressure. If this condition is not met, the currents in the propellant distort the magnetic field, and the present analysis is inadequate.

The second term in equation (9) represents the effect the fluid elements have on each other's motion through the exertion of pressure. The first factor, $p/\rho v_w^2$, in the term is the square of the ratio of the local speed of sound to the speed of the wave; it can be anticipated that this will be very small. The second factor, ρ/ρ_o , is on the average about v_o/v_w , also a small number. These factors should dominate the logarithmic derivative. Moreover, if the temperature throughout the gas does not vary too greatly from an average value T_a , it is approximately correct to set $p = \rho RT_a$ and to write for the pressure-gradient term (by using eq. (6))

$$\frac{p}{\rho v_w^2} \frac{\rho}{\rho_o} \frac{\partial}{\partial \alpha} \ln \frac{p}{p_o} = - \frac{RT_a}{v_w^2} \frac{\partial \xi}{\partial \alpha} - 2 \frac{\partial^2 \xi}{\partial \alpha^2}$$

An estimate of the dimensionless parameters that control the sizes of the various effects discussed previously can be obtained by taking the following typical operating conditions:

Propellant inflow, kg/sec	10^{-6}
Magnetic wavelength, m	1
Cross-sectional area, sq m	5×10^{-3}
Magnetic field maximum, Wb/sq m	0.03
Velocity of magnetic wave, m/sec	10^5
Conductivity, mho/m	10^4
Angular frequency of magnetic wave, rad/sec	$3\pi \times 10^5$
Initial density, kg/cu m.	10^{-5}
Molecular weight, kg/g mole (argon).	0.04
Average temperature, $^{\circ}\text{K}$	2000

Under these conditions

$$\kappa \equiv \frac{\sigma_o B_o^2}{\rho_o \omega} \approx 1 \quad (13)$$

$$\frac{\rho_o v_o v_w}{B_o^2 / \mu} \approx 0.03 \quad (14)$$

$$\epsilon \equiv \frac{RT_a}{v_w^2} \approx 10^{-4} \quad (15)$$

The interaction parameter κ (eq. (13)) has the appropriate size for effective acceleration. The condition of small Reynolds number as expressed in equation (12) is also met (eq. (14)), so there should not be large-scale distortions in the magnetic field, and equation (2) (or (3)) should suffice as a description of the field. Moreover, the pressure-gradient term has a very small coefficient ϵ (eq. (15)); this term can, therefore, be expected to remain small throughout the motion. A final observation is that, under the conditions given previously, the self-collision time of the atoms is comparable to or greater than the time they remain in the accelerator. Consequently, any energy added to the gas during the acceleration process will not be transformed to thermal motion of the atoms, and the assumption of a more or less constant temperature T_a should not cause great error.

It will be noted that the interaction parameter κ (eq. (13)) has been evaluated in terms of the conductivity σ_o and density ρ_o at the entrance to the accelerator rather than locally as required by condition (10). It is assumed hereafter in the analysis that the ratio of conductivity to density is a constant throughout the flow, or

$$\frac{\sigma}{\rho} = \frac{\sigma_0}{\rho_0} \quad (16)$$

This assumption is made to simplify the analysis - specifically, to help reduce the partial differential equation of motion to an ordinary differential equation. The effect of this assumption on the calculated motion of the fluid will be estimated only after an examination of the results.

With the assumption of equation (16) for the conductivity, the magnetic Reynolds number for the forgoing example is $\mu\sigma\lambda v_w \approx 0.25$. This estimate shows that a moderate amount of field distortion can be expected, and that distortion is more likely than would be surmised from equation (14) alone.

It is convenient to make one more change in the dependent variable to obtain a final form for the momentum equation. Let

$$\varphi(\alpha, \tau) \equiv \xi - \tau \quad (17)$$

where φ is the phase of the magnetic wave at time τ and position ξ of a particle. The introduction of equations (12), (15), and (17) into the momentum equation (9) yields

$$\frac{\partial^2 \varphi}{\partial \tau^2} + \kappa \frac{\partial \varphi}{\partial \tau} \sin^2 \varphi = \epsilon \frac{\frac{\partial^2 \varphi}{\partial \alpha^2}}{\left(\frac{\partial \varphi}{\partial \alpha}\right)^2} \quad (18)$$

The initial conditions accompanying this equation are found by considering the fluid particle that arrives at the origin ($\xi = 0$) at time t . Its particle number (initial location) is

$$a = -v_0 t$$

or

$$\alpha = -r\tau \quad (19)$$

$$r \equiv v_0/v_w$$

Equation (19) describes the initial curve in the α, τ -plane, along which the phase φ and its derivatives must be given:

$$\varphi = -\tau = \frac{\alpha}{r} \quad (20)$$

$$\frac{\partial \varphi}{\partial \tau} = \frac{\partial \xi}{\partial \tau} - 1 = r - 1 \quad (21)$$

$$\frac{\partial \varphi}{\partial \alpha} = \frac{\partial \xi}{\partial \alpha} = \frac{\rho_0}{\rho} = 1 \quad (22)$$

In equation (21), $\partial \xi / \partial \tau$ is the fluid velocity normalized to the wave velocity. The continuity equation (6) has been employed in equation (22). Equations (18) to (22) comprise the complete boundary-value problem for the system of fluid particles which is accelerated by the magnetic wave.

PERTURBATION ANALYSIS

Inasmuch as a small dimensionless number ϵ arises naturally in the formulation of the equation of motion (18), it is convenient to regard it as a perturbation parameter for reducing the partial differential equation to a system of ordinary equations. If the dependent variable φ is expanded in powers of ϵ ,

$$\varphi = \varphi^0 + \epsilon \varphi^1 + \dots \quad (23)$$

the coefficients of the first two powers in the momentum equation (18) are as follows:

For ϵ^0 :

$$\frac{\partial^2 \varphi^0}{\partial \tau^2} + \kappa \frac{\partial \varphi^0}{\partial \tau} \sin^2 \varphi^0 = 0 \quad (24)$$

For ϵ^1 :

$$\frac{\partial^2 \varphi^1}{\partial \tau^2} + \kappa \frac{\partial}{\partial \tau} (\varphi^1 \sin^2 \varphi^0) = \frac{\frac{\partial^2 \varphi^0}{\partial \alpha^2}}{\left(\frac{\partial \varphi^0}{\partial \alpha} \right)^2} \quad (25)$$

The corresponding initial conditions are

$$\left. \begin{aligned} \tau &= -\frac{\alpha}{r} \\ \varphi^0 &= \frac{\alpha}{r} \\ \frac{\partial \varphi^0}{\partial \tau} &= r - 1 \end{aligned} \right\} \quad (26)$$

$$\left. \begin{aligned} \varphi^1 &= 0 \\ \frac{\partial \varphi^1}{\partial \tau} &= 0 \end{aligned} \right\} \quad (27)$$

The zero-order equation of motion (24) represents the motion of one particle alone that is unaffected by the presence in the channel of other particles; this is the result of the disappearance of the pressure-gradient term to this order of magnitude. Physically, the meaning is that the magnetic wave moves so fast (with speed v_w) and accelerates the fluid so quickly that the forces due to pressure gradients, which propagate with the speed of sound, do not have time to readjust the fluid distribution during the acceleration process. Of course, this condition will no longer hold after the fluid has been accelerated to nearly the wave speed, for then $(\partial \varphi / \partial \tau) \approx 0$ and the Lorentz force term is negligible. The effects of pressure gradient will then take over.

The first-order equation (25) accounts for the effect of pressure gradient; that is, φ^1 makes a correction on φ^0 to account for the neglect of the pressure term in equation (24). Moreover, both equations (24) and (25) can now be regarded as ordinary differential equations in τ because the partial derivatives of any φ^n with respect to the material coordinate α are always transferred to the equation of order $n + 1$. The initial conditions (eqs. (20) and (22)) are satisfied entirely by φ^0 . The higher order φ^n have homogeneous initial conditions.

The first integral of the zero-order equation is (when using the notation φ_τ^0 for $\partial \varphi^0 / \partial \tau$)

$$\varphi_\tau^0(\tau, \alpha) = r - 1 - \frac{\kappa}{4} \left(2\varphi^0 - 2\frac{\alpha}{r} - \sin 2\varphi^0 + \sin 2\frac{\alpha}{r} \right) \quad (28)$$

The variable τ may be found as a function of φ^0 by integration from the initial condition (26). Then

$$\tau = -\frac{\alpha}{r} + \int_{\alpha/r}^{\varphi^0} \frac{dz}{\varphi_z^0(z; \alpha)} \quad (29)$$

Equation (29) specifies the time τ at which the α^{th} fluid element arrives at the location in the wave having phase φ^0 . The corresponding position and velocity are, to zero order in ϵ ,

$$\xi = \varphi^0 + \tau \quad (30)$$

and

$$v = v_w \xi_\tau \equiv v_w (\varphi_\tau^0 + 1) \quad (31)$$

The only remaining piece of information to be found is the density. When equation (29) is differentiated with respect to α and when the expression for the density (eq. (6)) is used, there results to lowest order in ϵ

$$\frac{\rho_0}{\rho} = \frac{\partial \xi}{\partial \alpha} = \varphi_\alpha^0 = \varphi_\tau^0 \left\{ \frac{1}{r-1} + \frac{\kappa}{r} \sin^2 \frac{\alpha}{r} \int_{\alpha/r}^{\varphi^0} \frac{dz}{[\varphi_z^0(z; \alpha)]^2} \right\} \quad (32)$$

This expression for the density, when evaluated on the initial curve, conforms to the initial condition (eq. (22)).

The first-order equation (25) with initial conditions (eq. (27)) can also be integrated directly. The first integral is

$$\varphi_\tau^1 + \kappa \varphi^1 \sin^2 \varphi^0 = \int_{\alpha/r}^{\varphi^0} \frac{\varphi_{\alpha\alpha}^0(z; \alpha)}{[\varphi_\alpha^0(z; \alpha)]^2} \frac{dz}{\varphi_z^0(z; \alpha)} \quad (33)$$

and the second integral is

$$\begin{aligned}
\varphi^1(\varphi^0; \alpha) = \exp \left[-\kappa \int_{\alpha/r}^{\varphi^0} \frac{\sin^2 z \, dz}{\varphi_z^0(z; \alpha)} \right] \\
\times \int_{\alpha/r}^{\varphi^0} \left\{ \int_{\alpha/r}^z \frac{\varphi_{\alpha\alpha}^0(u; \alpha) du}{\left[\varphi_\alpha^0(u; \alpha) \right]^2 \varphi_u^0(u; \alpha)} \right\} \exp \left[\kappa \int_{\alpha/r}^z \frac{\sin^2 u \, du}{\varphi_u^0(u; \alpha)} \right] \frac{dz}{\varphi_z^0(z; \alpha)} \quad (34)
\end{aligned}$$

For the explicit evaluation of the integrals in equations (33) and (34), it is necessary to have $\varphi_{\alpha\alpha}^0$ as well as the expressions (eqs. (28) and (32)) for φ_τ^0 and φ_α^0 . Differentiation of equations (28) and (32) yields

$$\begin{aligned}
\varphi_{\alpha\alpha}^0 = \kappa \left(\frac{1}{r} \sin^2 \frac{\alpha}{r} - \varphi_\alpha^0 \sin^2 \varphi^0 \right) & \left\{ \frac{1}{r-1} + \frac{\kappa}{r} \sin^2 \frac{\alpha}{r} \int_{\alpha/r}^{\varphi^0} \frac{dz}{\left[\varphi_z^0(z; \alpha) \right]^2} \right\} \\
& + \frac{\kappa}{r} \varphi_\tau^0 \left(\frac{1}{r} \sin \frac{2\alpha}{r} \int_{\alpha/r}^{\varphi^0} \frac{dz}{\left[\varphi_z^0(z; \alpha) \right]^2} + \sin^2 \frac{\alpha}{r} \left\{ \frac{\varphi_\alpha^0}{\left(\varphi_\tau^0 \right)^2} - \frac{1}{r(r-1)^2} \right. \right. \\
& \left. \left. - 2 \frac{\kappa}{r} \sin^2 \frac{\alpha}{r} \times \int_{\alpha/r}^{\varphi^0} \frac{dz}{\left[\varphi_z^0(z; \alpha) \right]^3} \right\} \right) \quad (35)
\end{aligned}$$

To first order, the position, density, and velocity of a particle can now be written in terms of φ^1 and its derivatives as

$$\xi = \varphi^0 + \epsilon \varphi^1 + \tau$$

$$\frac{\rho_0}{\rho} = \varphi_\alpha^0 + \epsilon \varphi_\alpha^1$$

$$\frac{v}{v_w} = 1 + \varphi_\tau^0 + \epsilon \varphi_\tau^1$$

EFFICIENCY AND THRUST

Energy is being added to the accelerated fluid in two ways: the kinetic energy is being increased by the acceleration process, and heat is being added by Joule dissipation. Since the propellant atoms are isothermal, the heat added must manifest itself in excitation and ionization of the fluid and in electron heating. When it is assumed that the heat added is sufficient to maintain the conductivity of the propellant and when the loss of kinetic energy by drag at the walls is neglected, the efficiency is

$$\eta = \frac{\text{kinetic energy added}}{\text{kinetic energy added} + \text{Joule heat added}} \quad (36)$$

The kinetic energy added per unit mass is

$$\frac{1}{2} (v^2 - v_o^2) = \frac{1}{2} v_w^2 (\xi_\tau^2 - r^2) = \frac{1}{2} v_w^2 \left[(\varphi_\tau^0 + 1)^2 - r^2 + \epsilon 2\varphi_\tau^1 (\varphi_\tau^0 + 1) + \dots \right]$$

The Joule heat per unit mass, $J^2/\sigma\rho$, when integrated over a particle trajectory is

$$\begin{aligned} \int_{-\alpha/r}^t \frac{\sigma}{\rho} (v_w - v)^2 B^2 dt &= v_w^2 \kappa \int_{\alpha/r}^\varphi \varphi_\tau \sin^2 \varphi d\varphi = v_w^2 \kappa \int_{\alpha/r}^{\varphi^0} \varphi_\tau^0 \sin^2 \varphi^0 d\varphi^0 \\ &+ \epsilon v_w^2 \kappa \int_{\alpha/r}^{\varphi^0} \left(\varphi_\tau^1 \sin^2 \varphi^0 + \varphi^1 \varphi_\tau^0 \sin 2\varphi^0 + \varphi_\tau^0 \sin^2 \varphi^0 \frac{\partial \varphi^1}{\partial \varphi^0} \right) d\varphi^0 + \epsilon v_w^2 \kappa \varphi^1 \varphi_\tau^0 \sin^2 \varphi^0 \end{aligned} \quad (37)$$

To zero order in ϵ ,

$$\eta = \frac{(\varphi_{\tau}^0 + 1)^2 - r^2}{(\varphi_{\tau}^0 + 1)^2 + 2\kappa \int_{\alpha/r}^{\varphi^0} \varphi_{\tau}^0 \sin^2 \varphi^0 d\varphi^0 - r^2}$$

After the integration in the denominator is performed, it is found that

$$\eta = \frac{\varphi_{\tau}^0 + 1 + r}{2} \equiv \frac{\xi_{\tau} + r}{2} \quad (38)$$

The efficiency to order zero associated with a fluid particle is simply the average of the final and initial dimensionless velocities of the particle. Since the dimensionless velocity has an upper limit of unity and r is very small, the efficiency has an upper limit of about 1/2, a result previously obtained by other investigators (ref. 8).

An average efficiency may be obtained for any station along the accelerator by averaging over all particles that enter during a half cycle of the magnetic wave. Thus,

$$\bar{\eta} = \frac{1}{2} (\bar{\xi}_{\tau} + r)$$

where

$$\bar{\xi}_{\tau}(\xi) \equiv \frac{1}{\pi} \int_{-\pi}^0 \xi_{\tau} \left(\xi, \frac{\alpha}{r} \right) d\left(\frac{\alpha}{r} \right)$$

The thrust of the accelerator is calculated by finding the total force acting on the fluid at any one time and then averaging over a period. If X is the length of the accelerator and C is its cross section, the force acting at any time is

$$F = \int_0^X \sigma(v_w - v) B^2 C dx = -\sigma_0 v_0 B_0^2 C k^{-1} \int_{\alpha/r=-\tau}^{\alpha(X)/r} \varphi_{\tau} \sin^2 \varphi d\left(\frac{\alpha}{r} \right)$$

The average of this force over a period of the magnetic field is the thrust

$$\Theta = \frac{1}{2\pi} \int_{\tau_0}^{\tau_0+2\pi} F(\tau) d\tau$$

Because the force is periodic it is permissible to change the area of integration in the α, τ -plane to represent the integration over the paths of a set of particles which come into the accelerator during a half cycle of the magnetic field. Then

$$\Theta = \frac{\sigma_o v_o B_o^2 C}{\pi k} \int_{\alpha/r=0}^{-\pi} d\left(\frac{\alpha}{r}\right) \int_{\tau=-\alpha/r}^{\tau(X)} \varphi_{\tau} \sin^2 \varphi d\tau = \frac{\sigma_o v_o B_o^2 C}{\kappa k} \frac{1}{\pi} \int_{-\pi}^0 (\xi_{\tau}(X) - r) d\left(\frac{\alpha}{r}\right)$$

Equation (28) has been used to obtain the last integral. If the definition of κ is employed, then

$$\Theta = \rho_o v_o C v_w (\bar{\xi}_{\tau}(X) - r)$$

Thus, the thrust is just the mass flow into the accelerator times the average increment of specific impulse, a result which bears a close resemblance to that for a steady flow. It should be emphasized, however, that the average appearing here is not a time average, but an average over fluid elements of equal mass.

NUMERICAL RESULTS

The integrals appearing in equations (29) and (32) for the zero-order motion of 50 fluid elements in a wave half cycle have been performed. These particles enter the accelerator at uniformly spaced times of $-\alpha/r = 0, \pi/50 \dots \pi$. Along with equa-

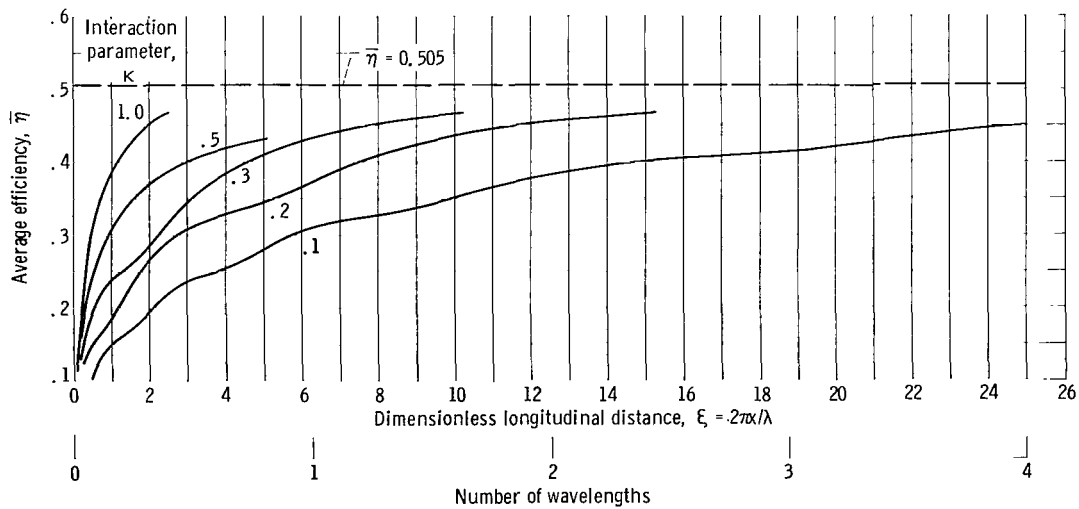


Figure 2. - Average efficiency of traveling-wave accelerator. Velocity ratio, v_o/v_w , 0.01.

tion (28) these integrations make it possible to compute the zero-order velocity, efficiency, mass density, and current density associated with each element of fluid. Inasmuch as efficiency and thrust are linearly related according to

$$\Theta = 2\rho_0 v_0 v_w C(\bar{\eta} - r)$$

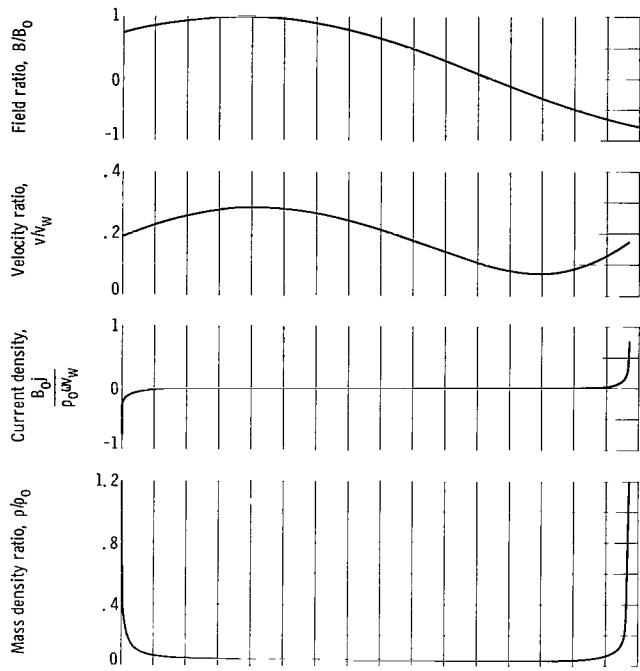
it is sufficient to work with the efficiency.

Figure 2 shows how the efficiency improves with length of run down the channel for various interaction parameters κ . The waviness in the curves can be attributed to the periodic nature of the driving force. Generally, an accelerator can be expected to be an integral number of waves in length in order that the driving coils can properly establish the traveling wave. For a one-wavelength accelerator, a strong interaction parameter such as $\kappa = 1.0$ seems to be greater than required to successfully accelerate the fluid in the single wavelength. At the other extreme, a value $\kappa = 0.1$ does not provide enough interaction between wave and fluid to get the fluid near the wave speed at the exit. For a one-wavelength accelerator, it appears that some intermediate value of the interaction parameter, say $\kappa = 0.3$ to 0.5 , would be appropriate.

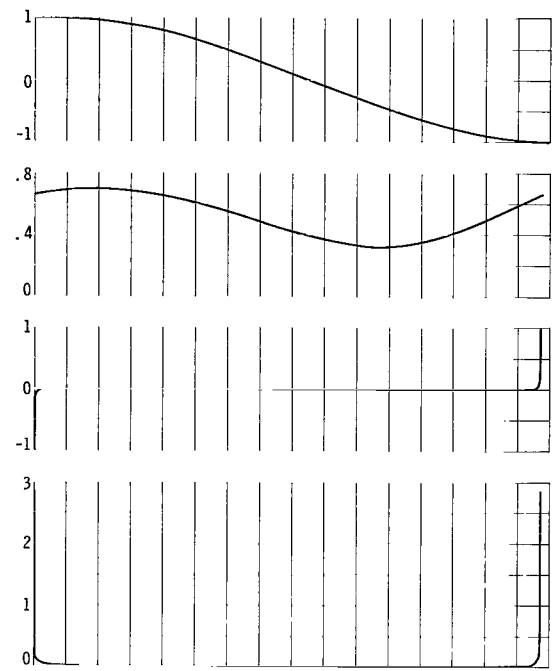
Figure 3 shows how the field, velocity, current density, and mass density vary in time at various stations and interaction parameters. The most striking features of these distributions are the sharp accumulations of mass and current. The peaking of these quantities becomes more severe as the fluid moves down the accelerator. Only when the fluid has nearly reached the wave velocity will pressure effects, as represented in first-order calculations, take over and redistribute the fluid in a more uniform manner in the beam. The ratio of the mass density in the peak to the mass density midway between peaks can vary over several orders of magnitude; values range from 30 to 3000 in the numerical examples computed. The fluid elements, which enter the accelerator chamber coincident with a node in the magnetic wave, form the centers of the regions of high density.

Although the first-order function φ^1 has not been calculated, the right side of equation (25), which determines its magnitude, has been computed at every step in the calculations. This term, which may be regarded, alternatively, as the neglected term of the zero-order equation (24), is generally of order unity, except possibly when a fluid element enters the accelerator in which case it may be as high as 90. This is still well within the allowable range set by the estimate of ϵ (eq. (15)). Therefore the zero-order solution should be a good approximation to the complete solution; the pressure effects are not really important.

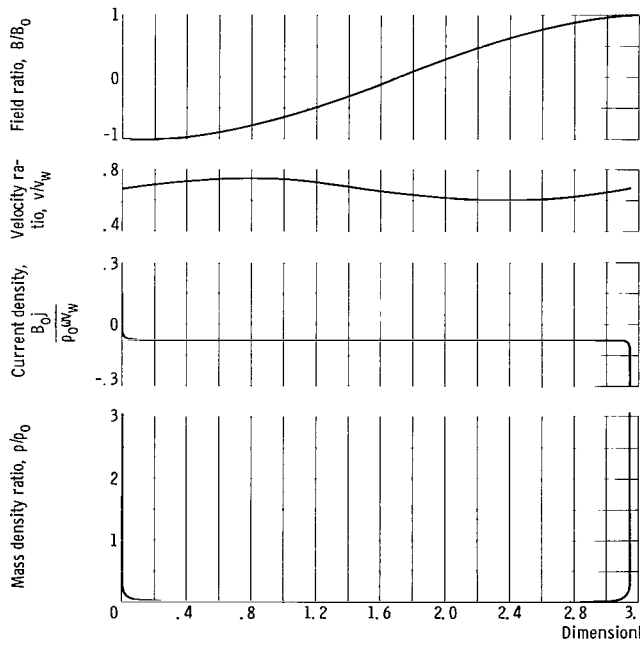
A comparison of the present work with the computations of Fabri and Moulin (ref. 6) is made in figures 4 and 5. There is a limit to the aptness of such a comparison because the analysis of Fabri and Moulin is valid only when the difference between initial fluid



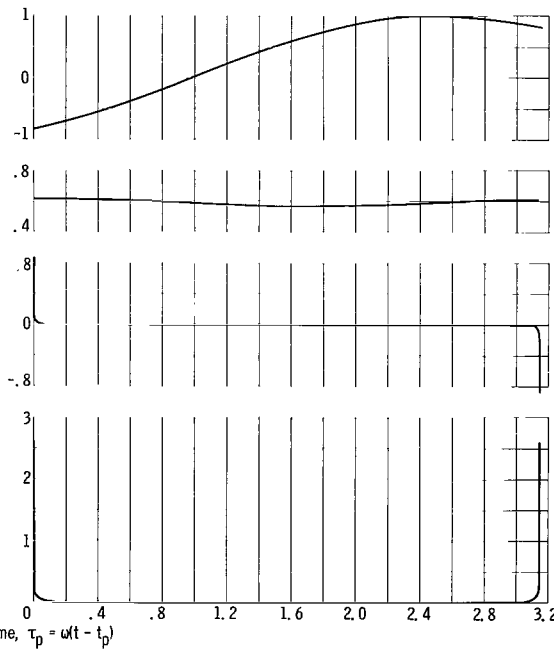
(a) Interaction parameter, κ , 1.0; dimensionless longitudinal coordinate, ξ , 0.05; velocity ratio, v_0/v_W , 0.01.



(b) Interaction parameter, κ , 1.0; dimensionless longitudinal coordinate, ξ , 0.5; velocity ratio, v_0/v_W , 0.01.



(c) Interaction parameter, κ , 0.3; dimensionless longitudinal coordinate, ξ , 3.0; velocity ratio, v_0/v_W , 0.01.



(d) Interaction parameter, κ , 0.1; dimensionless longitudinal coordinate, ξ , 6.0; velocity ratio, v_0/v_W , 0.01.

Figure 3. - Time variation of field and fluid properties in traveling-wave accelerator.

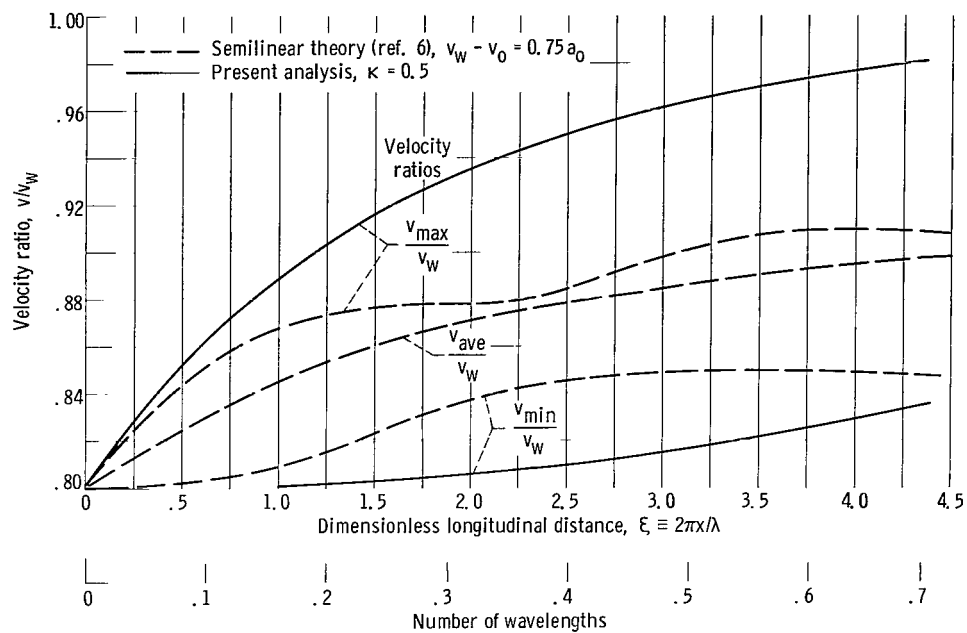


Figure 4. - Velocity extrema in an accelerator. Velocity ratio, v_0/v_w , 0.8.

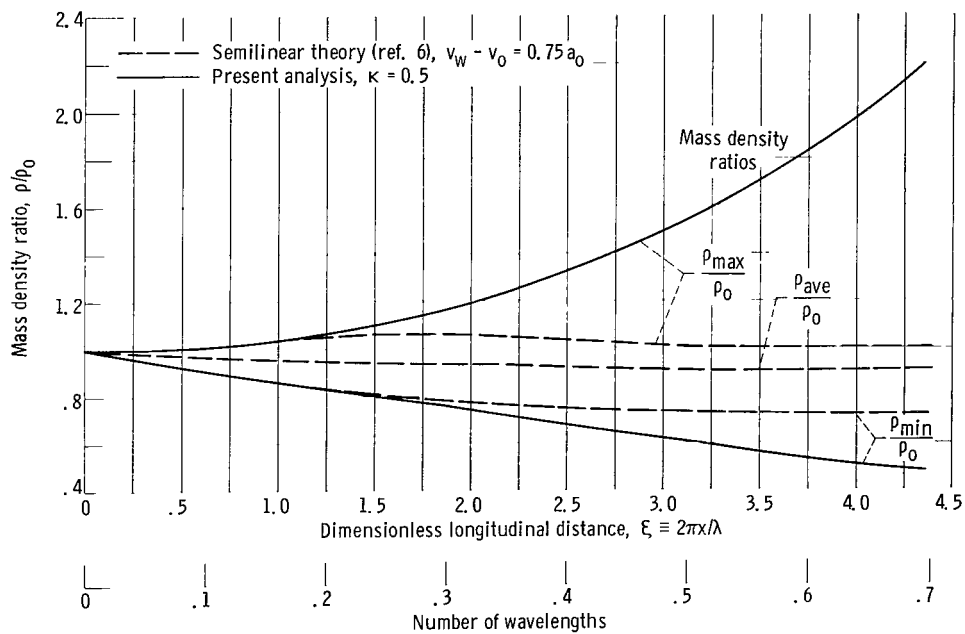


Figure 5. - Density extrema in an accelerator. Velocity ratio, v_0/v_w , 0.8.

velocity and magnetic wave velocity is small, whereas the present analysis holds when the magnetic wave velocity is much greater than the speed of sound and, therefore, the inlet velocity. The present analysis, consequently, drops the pressure-gradient term. The example used for comparison is in the range of inlet velocities for which the assumptions at Fabri and Moulin are valid. Nevertheless, the results of the two computations show much similarity. Figure 4 shows maximum and minimum velocities observed at any station along the accelerator, and figure 5 shows maximum and minimum densities. The two solutions yield much the same variation of the extreme values near the inlet, but further downstream the effect of the pressure-gradient term is very apparent in the solution of Fabri and Moulin. The relatively narrow bands of velocities and densities in that solution must be due principally to the tendency of the pressure gradient to keep the flow uniform. Although the time-average velocity has not been computed in the present computations, it appears from figure 4 that the average velocities for the two methods should be in good agreement.

The curves of figures 4 and 5 also show that the maximum velocities and densities for the two methods differ more in the cited example than do the minimum values. This fact may be associated with the assumption in the present work that the conductivity is proportional to the density, as contrasted to the assumption of Fabri and Moulin of constant conductivity, for the former assumption leads to regions of high coupling between fluid and wave wherever increases in density occur. Eventually a large fraction of the fluid accumulates in high-density regions which are moved close to wave speed. If the minimum velocities and densities for the two methods are generally in more agreement, as is indicated by this one example, then the present method will give a higher estimate of the efficiency of the acceleration process than would be expected under an assumption of constant conductivity.

In considering the previous example as described by the two methods of calculation, it should be kept in mind that the example is not a very realistic one because of the small differences in velocity between wave and fluid. It is chosen to make comparison easy with the method of Fabri and Moulin. The present method, valid for large velocity differences, quite correctly ignores pressure gradients under practical conditions.

One other question that arises in connection with the assumption of conductivity proportional to density is whether the Joule heating can provide sufficient energy to keep the gas in a state of high conductivity as the postulate requires. A simple calculation suffices to show that it probably can. From equation (37) the energy dissipated per unit mass over the time the gas is in the accelerator is of order $v_w^2/10$. For argon this is equivalent to about 40 electron volts per atom. Since most of the dissipation occurs near the entrance and the decay time for free electrons in argon is 10^{-3} second as opposed to 10^{-5} second acceleration time, there appears to be enough energy to keep a sufficient

fraction (say 1 percent) of the atoms ionized and to raise the electron temperature, even if excitation of atoms absorbs a large amount.

LONGITUDINAL-WAVE ACCELERATOR

Another form of traveling-wave accelerator uses a moving magnetic field with a potential

$$A = - \frac{B_0}{k} \sin \varphi \sinh ky$$

as contrasted to equation (2). The field resembles qualitatively that produced by coils wrapped around the channel at equal intervals Δx and having equal phase differences. In addition to the traveling magnetic wave, a uniform field B_s parallel to the direction of motion may be impressed to help contain the fluid and keep it from the walls. The equations of motion are, neglecting pressure gradients,

$$\frac{\rho}{\sigma} \ddot{x} = - (\dot{x} - v_w) B_y^2 + \dot{y} B_x B_y$$

$$\frac{\rho}{\sigma} \ddot{y} = (\dot{x} - v_w) B_x B_y - \dot{y} B_x^2$$

and the initial conditions on each fluid element are

$$\dot{\varphi} = r - 1$$

$$\dot{\xi} = 0$$

The components of magnetic field are

$$B_x = - B_0 \sin \varphi \cosh ky + B_s$$

$$B_y = B_0 \cos \varphi \sinh ky$$

In terms of the definitions,

$$\beta = \frac{B_s}{B_o}$$

$$\zeta = ky$$

the dimensionless equations of motion are, for small ζ ,

$$\ddot{\varphi} = -\kappa \left[\dot{\varphi} \cos^2 \varphi \sinh^2 \zeta + \dot{\zeta} (\sin \varphi \cosh \zeta - \beta) \cos \varphi \sinh \zeta \right] \quad (39)$$

$$\ddot{\zeta} = -\kappa \left[\dot{\varphi} (\sin \varphi \cosh \zeta - \beta) \cos \varphi \sinh \zeta + \dot{\zeta} (\sin \varphi \cosh \zeta - \beta)^2 \right] \quad (40)$$

The efficiency for a particle is given by

$$\eta = \frac{(\dot{\varphi} + 1)^2 - r^2}{\frac{2H}{v_w^2} + (\dot{\varphi} + 1)^2 + \dot{\zeta}^2 - r^2}$$

where

$$H \equiv \kappa v_w^2 \int \left[\dot{\varphi} \cos \varphi \sinh \zeta + \dot{\zeta} (\sin \varphi \cosh \zeta - \beta) \right]^2 d\tau$$

The equations of motion (39) and (40) have been integrated for two cases - one with a confining field ($\beta = 1$) and one without ($\beta = 0$). In each case the calculations have been terminated at a lateral position designated as the "wall"; that is, no particles are considered which enter at a distance farther from the center line than the wall position, and the trajectory of any particle is no longer followed after it crosses the wall ordinate.

In the first of these examples, the width of the accelerator is about $\lambda/10$. Trajectories have been obtained for fluid elements entering at both the wave node and the wave crest. The lateral displacements of the elements at the entrance are $\zeta = 0.01, 0.02, \dots, 0.29, 0.30$. For the second example, an accelerator with no confining field, the width is approximately 0.6λ and the entrance ordinates are $\zeta = 0.1, 0.2, \dots, 1.8$. The width, in this case, corresponds to the optimum coil radius of 0.3λ found by Palmer, Jones, and Seikel (ref. 11) for the efficient production of a traveling wave by a system of equally spaced circular coils. In the second example, also, trajectories for fluid elements entering at the wave crest and node have been found. For both cases the interaction parameter is $\kappa = 1$ and the velocity ratio is $r = 0.01$.

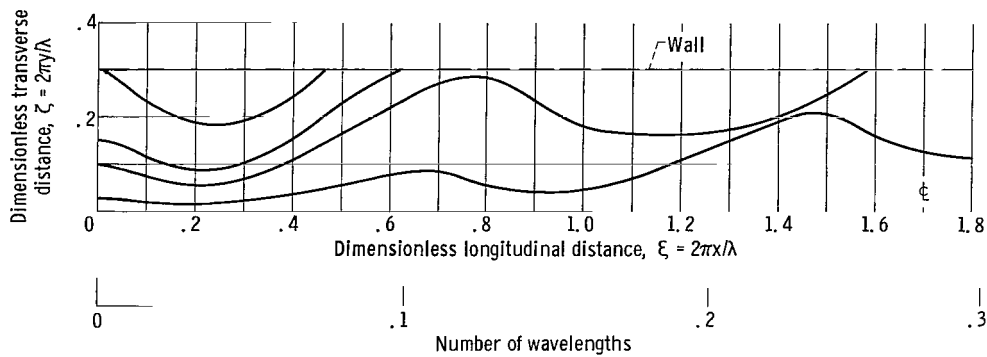


Figure 6. - Propellant trajectories in longitudinal wave accelerator. Interaction parameter, κ , 1; magnetic field ratio, B_s/B_0 , 1; velocity ratio, v_0/v_W , 0.01.

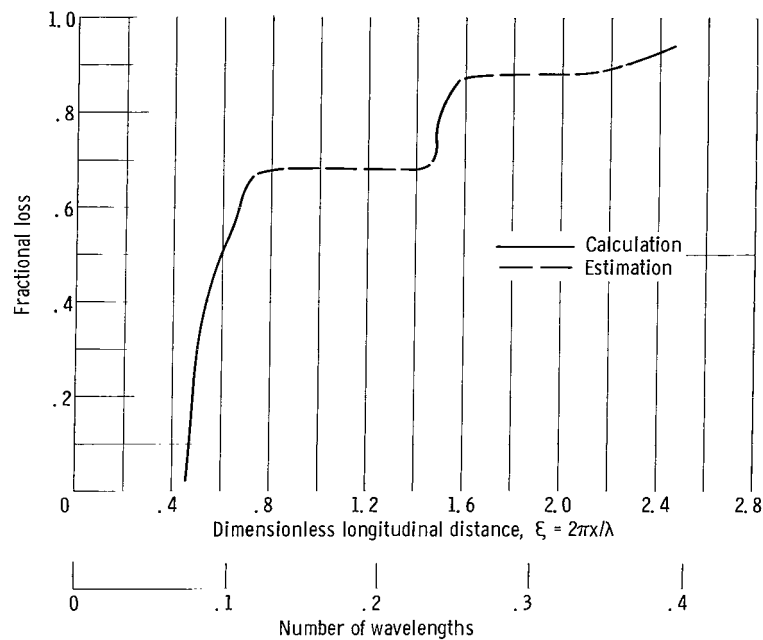


Figure 7. - Loss of propellant entering at wave node to walls of longitudinal wave accelerator. Phase of magnetic wave, ϕ , zero when dimensionless longitudinal distance, ξ , is zero; accelerator width, approximately one-tenth wavelength of magnetic wave, λ ; interaction parameter, κ , 1; magnetic field ratio, B_s/B_0 , 1; velocity ratio, v_0/v_W , 0.01.

Figure 6 shows propellant trajectories in the first case when there is a confining field. It is apparent that eventually all fluid elements will strike the walls. A large fraction is lost even on the first undulation, which occurs within one-tenth of a wavelength from the entrance. This effect is displayed better in figure 7, which shows the fractional loss as a function of accelerator length. It is apparent that the inclusion of a moderate ($\beta = 1$) confining field cannot prevent the loss to (or through) the boundaries of the accelerator of a sizable fraction of the fluid. The efficiencies associated with the fluid elements at the time they hit the wall are 15 percent or less. Calculations for fluid elements entering at the wave crest show essentially the same behavior (fig. 8),

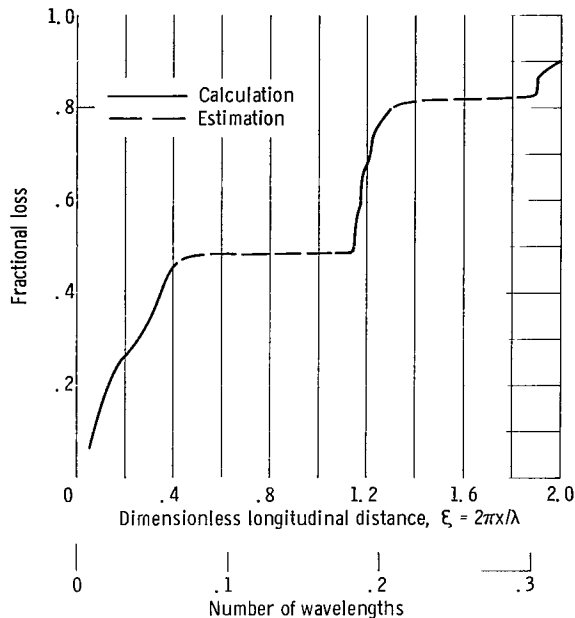


Figure 8. - Loss of propellant entering at wave crest to walls of longitudinal wave accelerator. Phase of magnetic wave, φ , $\pi/2$ when dimensionless longitudinal distance, ξ , is zero; accelerator width, approximately one-tenth wavelength of magnetic wave, λ ; interaction parameter, κ , 1; magnetic field ratio, B_s/B_0 , 1; velocity ratio, v_0/v_w , 0.01.

except that they are thrown to the walls roughly a quarter of a cycle earlier.

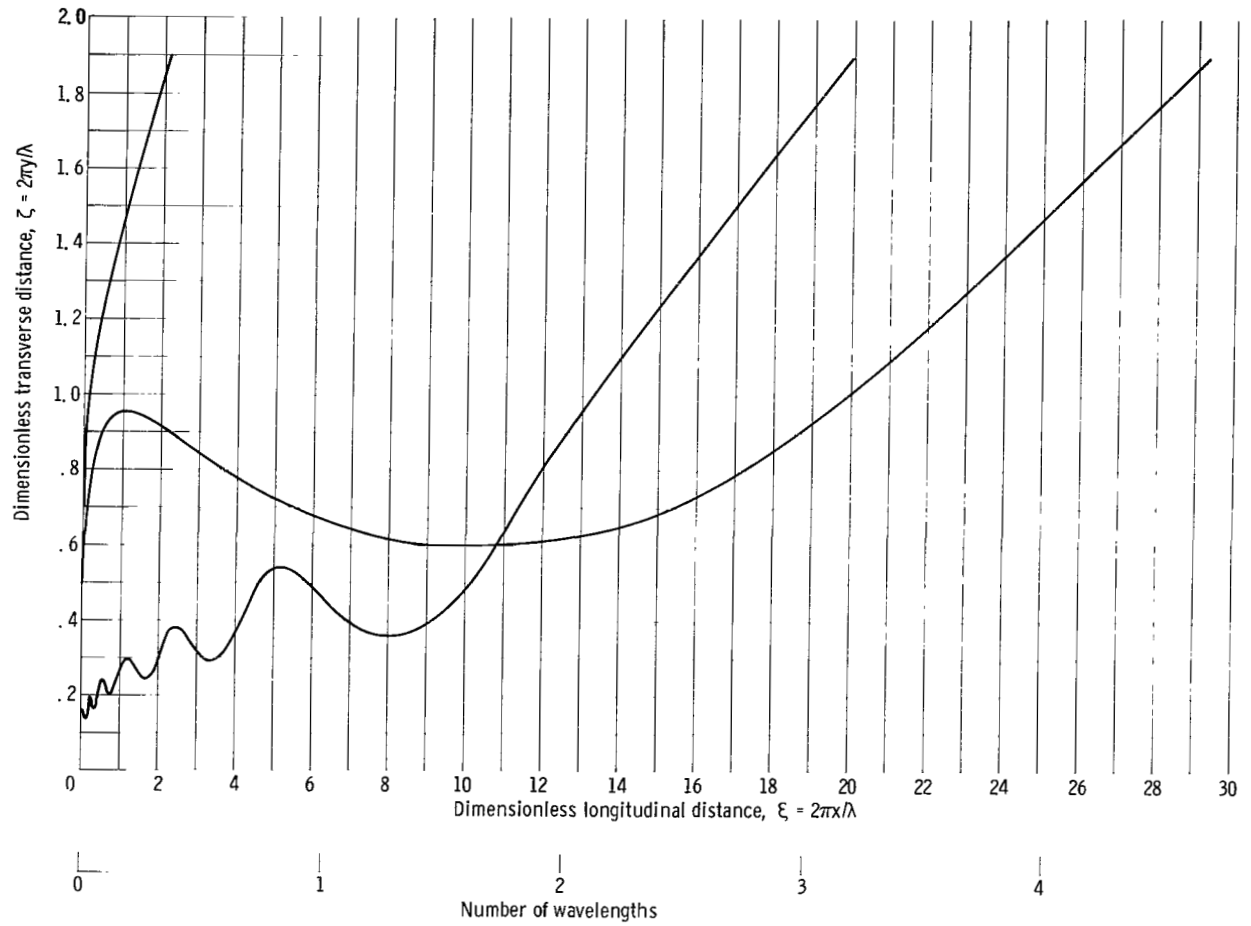
When the accelerator channel is wide, as specified in the previous second example, the trajectories of the fluid elements show a considerable variation, which depends on the entrance coordinate ζ . Some examples are given in figures 9(a) and (b). The time that each particle is in the accelerator before striking the wall can be estimated by the number of undulations it undergoes. Each rise or fall represents the passage of one-fourth wavelength of the magnetic field and, therefore, a quarter period of time. Although a fluid element close to the centerline may reach the wall at an earlier axial position than an element further out, it does so at a later time. An interesting feature of the trajectories shown is that they all terminate in a portion that is nearly rectilinear motion.

Moreover, this motion is found to take place

at nearly constant velocity. In this last portion of its flight, each fluid element affixes itself to a magnetic field line and rides out of the accelerator at constant velocity, much as was the case in the normal-field accelerator, but in this case the velocity is not parallel to the accelerator axis.

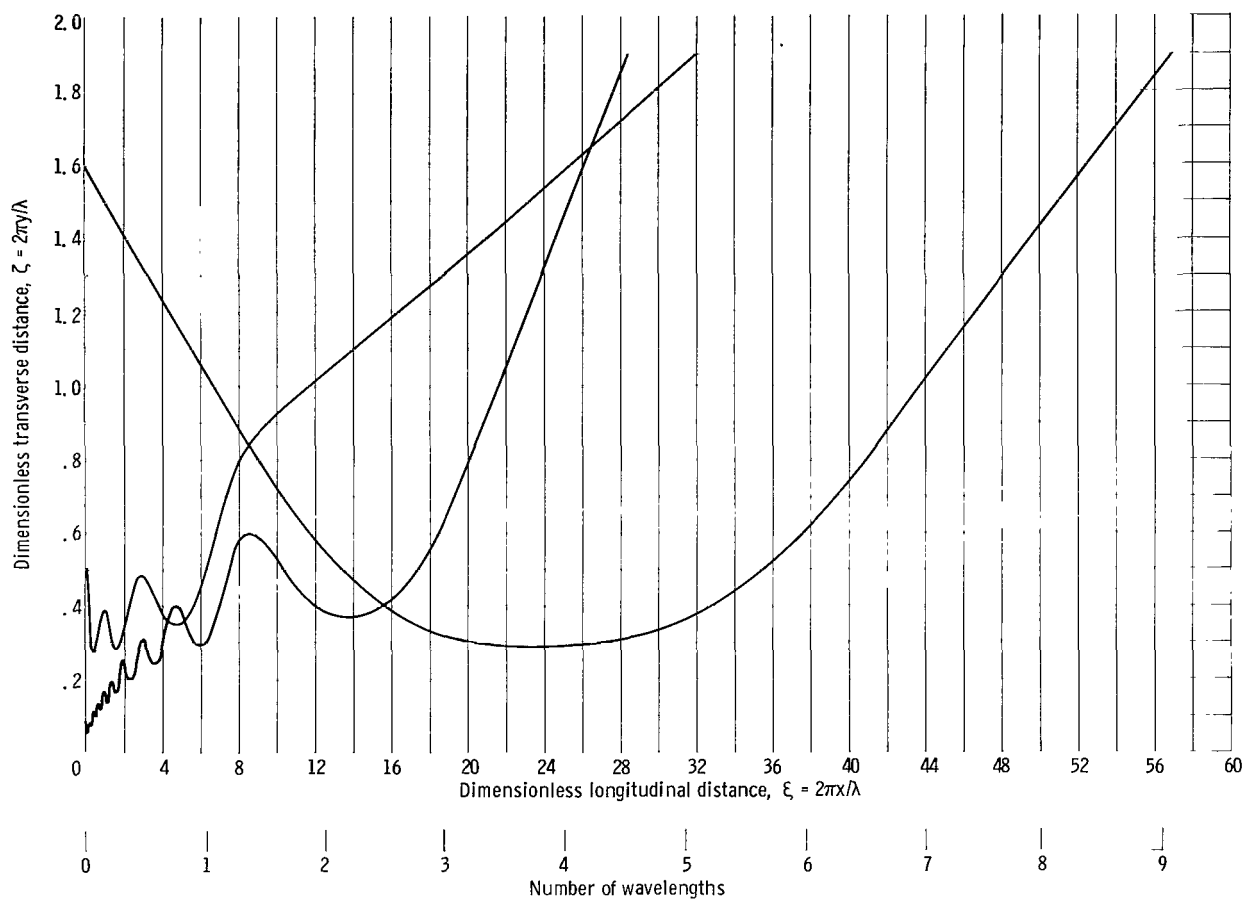
Figures 10 and 11 show the fractional loss to the walls for fluid entering at the wave nodes and crests, respectively. There are large losses to the walls again; however, the average efficiency of the particles is higher than in the narrow channel case. For example, the fluid elements that reach an axial position of one wavelength ($\xi = 2\pi$) have an average efficiency of about 0.4, and, if no contribution is allowed from those elements which strike the wall, the average efficiency for all is about 0.26. This is a considerable improvement over the previous case and reinforces the findings of Palmer, Jones, and Seikel (ref. 11) that the wider accelerating channel should be the more efficient one.

Finally, it appears that a confining field is less effective in containing the propellant than is the widening of the channel. A comparison of figures 7 and 10 and figures 8 and 11 shows that most of the propellant in the narrow accelerator having a confining field is lost within a wavelength, whereas in the wider channel having no confining field a correspondingly high level of loss is not reached for several wavelengths.



(a) Particles entering at maximum longitudinal field. Phase of magnetic wave, φ , $\pi/2$ when dimensionless longitudinal distance, ξ , is zero.

Figure 9. - Propellant trajectories in longitudinal wave accelerator. Accelerator width, approximately six-tenths wavelength of magnetic wave, λ ; interaction parameter, κ , 1; magnetic field ratio, B_s/B_0 , 0; velocity ratio, v_0/v_w , 0.01.



(b) Particles entering at minimum longitudinal field. Phase of magnetic wave, φ , zero when dimensionless longitudinal distance, ξ , is zero.

Figure 9. - Concluded.

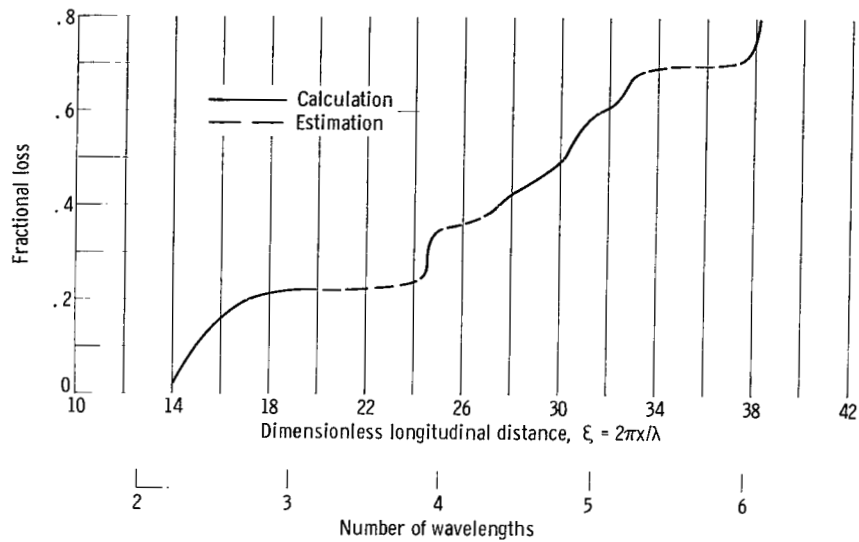


Figure 10. - Loss to walls of propellant entering at minimum longitudinal field. Phase of magnetic wave, φ , zero when dimensionless longitudinal distance, ξ , is zero; accelerator width, approximately six-tenths wavelength of magnetic wave, λ ; interaction parameter, κ , 1; magnetic field ratio, B_s/B_0 , 0; velocity ratio, v_0/v_w , 0.01.

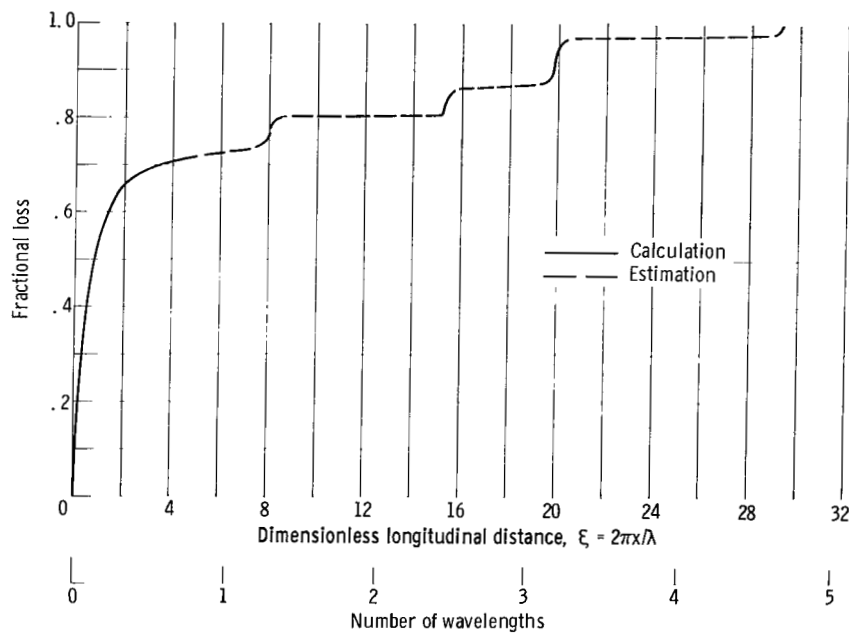


Figure 11. - Loss to walls of propellant entering at maximum longitudinal field. Phase of magnetic wave, φ , $\pi/2$ when dimensionless longitudinal distance, ξ , is zero; accelerator width, approximately six-tenths wavelength of magnetic wave, λ ; interaction parameter, κ , 1; magnetic field ratio, B_s/B_0 , 0; velocity ratio, v_0/v_w , 0.01.



CONCLUDING REMARKS

Principally, the limits on the accuracy of the analysis made here come from the assumption that the conductivity is proportional to the density. The assumption was made for convenience in reducing the equation to manageable form, and it is difficult to say just how the conductivity should be represented in an unsteady, nonuniform situation as is presented in the accelerator. The most that can be said of the effect of the conductivity assumption is that it probably exaggerates the variations of density and velocity that occur in the flow, although the average values should be representative. In this connection, the upper limit of 0.5 on the efficiency is a good indication of the performance of the normal-field accelerator.

The solutions to the longitudinal-wave accelerator have a further limitation in the amount of information they can offer because it is not possible to obtain the density without first finding the trajectories of a very great many fluid elements. Nevertheless, the relative efficiencies for the two calculated cases - narrow and wide channel - are sufficient to illustrate that accelerators of greater ratio of coil radius to wavelength are more efficient, which is just a reflection of the increased efficiency of acceleration of a fluid element with the starting value of y or ζ , and the attendant increase in the normal component of field. Even the wide-channel accelerator of this type, however, attains only about one-half (0.26) the efficiency reached by the normal field configuration, and then only if the accelerator is several wavelengths long.

REFERENCES

1. Klein, Milton M.; and Brueckner, Keith A.: Plasma Propulsion by a Rapidly Varying Magnetic Field. *J. Appl. Phys.*, vol. 31, no. 8, Aug. 1960, pp. 1437-1448.
2. Matthews, Clarence W.: A Theoretical Study of the Motion of an Idealized Plasma Ring Under the Influence of Various Coaxial Magnetic Fields. NASA TR R-121, 1961.
3. Matthews, Clarence W.: A Theoretical Study of the Motion of an Idealized Plasma Ring Through Various Traveling-Magnetic-Wave Plasma Accelerators. NASA TN D-1636, 1963.
4. Meyer, Rudolph X.: Magnetic Plasma Propulsion by Means of a Traveling Sinusoidal Field. Rept. No. TR 60-0000-09114, Space Tech. Labs., Apr. 30, 1960.
5. Covert, Eugene E.; and Haldeman, Charles W.: A One-Dimensional Model for Compressible Flow in the Traveling Wave Pump. Rept. No. ARL-63-83, M. I. T., May 1963.
6. Fabri, Jean; and Moulin, Thiebault: Action d'un champ glissant sur un écoulement conducteur de l'électricité. Eleventh Congrès Int. de Mécanique Appliquée (ONERA TP No. 163), Munich, West Germany, Aug. 30-Sept. 5, 1964.
7. Bernstein, I. B.; Kulsrud, R. M.; Fanucci, J. B.; Fishbeck, K. H.; Jarem, J.; Korman, N. I.; Ness, N.; and Lessen, M.: An Electrodeless MHD Generator. *Eng. Aspects of Magnetohydrodynamics*, C. Mannal and N. W. Mather, eds. Columbia Univ. Press, 1962, pp. 255-276.
8. Heflinger, Lee; and Schaffer, Allan: Magnetic Induction Plasma Engine, Summary Report. NASA CR 54063, 1964.
9. Jones, Robert E.; and Palmer, Raymond W.: Traveling Wave Plasma Engine Program at NASA Lewis Research Center. Third Symposium on Eng. Aspects of Magnetohydrodynamics, AIEE, IAS, IRE, Univ. Rochester, Mar. 28-30, 1962.
10. Lamb, Horace: *Hydrodynamics*. Dover Pub., 1945, p. 13.
11. Palmer, Raymond W.; Jones, Robert E.; and Seikel, George R.: Analytical Investigations of Coil-System Design Parameters for a Constant-Velocity Traveling Magnetic Wave Plasma Engine. NASA TN D-2278, 1964.

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